Class 11 Mathematics Project

Topic: Identify Distinction Between a Relation and a Function with Suitable Examples and Graphical Illustration

Submitted By:
Name:
Class: XI
Subject: Mathematics
School:
Academic Year:

Preface

This project on “Distinction Between a Relation and a Function” helped me understand how elements from two sets are connected using relations, and how functions are special types of relations. I have explained the topic using basic definitions, examples, graphs, and the Vertical Line Test.

With the help of mapping diagrams and the Vertical Line Test, I was able to show which relations qualify as functions and which do not. This project improved my understanding and helped me present mathematical ideas clearly.

I am thankful to my Mathematics teacher for providing guidance and to my school for giving me this opportunity to engage with such an enriching mathematical concept.

INDEX

---

1. Introduction

In mathematics, the concepts of relation and function form the foundation for understanding how two sets of values can be connected logically. A relation describes any rule or correspondence that pairs elements of one set with elements of another set. In contrast, a function is a specific type of relation that assigns exactly one output to each input in the domain.

Understanding the difference between a general relation and a function is essential in many areas of mathematics and its applications, including algebra, coordinate geometry, and real-life problem-solving. A function must satisfy the condition that no single input maps to more than one output. This property can also be tested graphically using the Vertical Line Test — a key visual tool used in this project.

This project aims to explore and explain the distinction between relations and functions through definitions, Venn diagrams, graphical plots, vertical line test and real-life examples. The goal is to provide a deeper understanding of how these mathematical ideas work and why the difference matters.

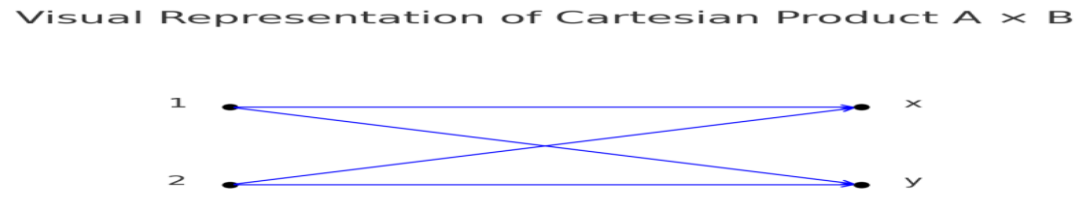
2. Objectives

The main objective of this project is to help students understand the basic definitions of sets, Cartesian products, relations, and functions, and to clearly distinguish between a relation and a function using suitable examples and illustrations. It aims to show the difference with mapping diagrams, Venn diagrams, and the Vertical Line Test, and to explain how only functions assign exactly one output to each input. The project also seeks to connect these concepts with real-life examples, develop analytical thinking, and provide a clear summary of the differences between a relation and a function.

3. Definition of Set and Cartesian Product

Set: A set is a well-defined collection of distinct elements. It is usually represented within curly braces.
Example: A = {1, 2, 3}

Cartesian Product: If A and B are two sets, then the Cartesian Product A × B is the set of all ordered pairs (a, b) where a ∈ A and b ∈ B.
Example: If A = {1, 2} and B = {x, y}, then:
A × B = {(1, x), (1, y), (2, x), (2, y)}



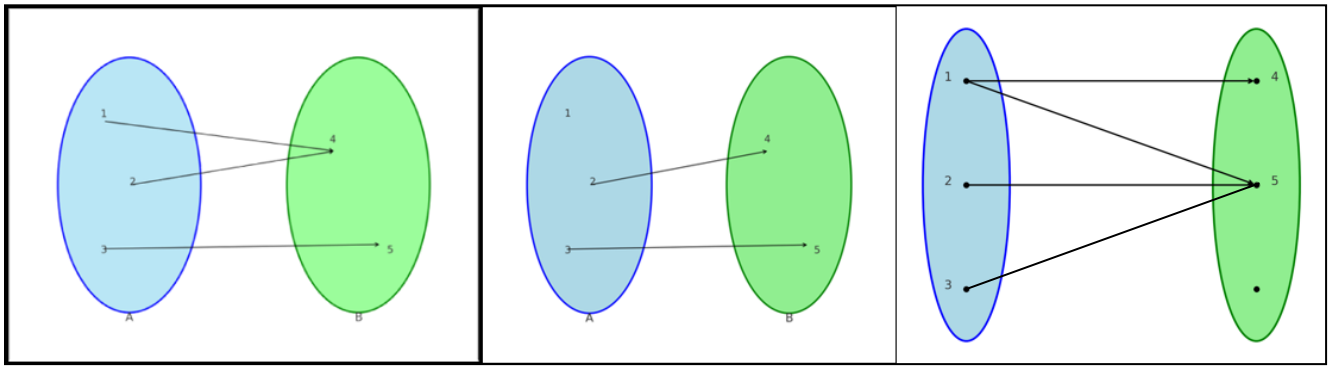
4. Definition of Relation with Example

A relation from a set A to a set B is a rule that associates each element of set A with one or more elements of set B. It is usually defined as a set of ordered pairs.

Mathematically:
If A and B are two non-empty sets, then a relation R from A to B is a subset of the Cartesian product A × B.
R = {(x, y) | x ∈ A, y ∈ B, x R y}

Example: Let A = {1, 2, 3}, B = {4, 5}.
Then, the following are some relations from A to B:
R_1 = {(1, 4), (2, 4), (3, 5)}
R_2 = {(2, 4), (3, 5)}
R_3 = {(1, 5), (1, 4), (2, 5), (3, 5)}

VENN DIAGRAM



5. Definition of Function

A function is a special type of relation in which each element of the domain is related to exactly one element of the codomain.

Mathematically:
A function f from set A to set B is a relation such that for every a ∈ A, there exists a unique b ∈ B such that (a, b) ∈ f.

Example:
Let A = {1, 2, 3}, B = {4, 5, 6}
Then, f = {(1, 4), (2, 5), (3, 6)} is a function.

Mathematical Clarification of Relations and Functions using Venn Diagrams:

1. First Diagram (Relation - Function):
The relation is R = {(1, 4), (2, 4), (3, 5)}
Each element of set A maps to exactly one element in set B. Hence, it satisfies the condition of a function.
Mathematically: $\forall a \in A, \exists b \in B$ such that $(a, b) \in R$

2. Second Diagram (Relation - Not a Function):
The relation is R = {(2, 4), (3, 5)}
Element '1' from set A has no mapping, which violates the function rule.
Mathematically: $\exists a \in A$ such that $\nexists b \in B$ with $(a, b) \in R$

3. Third Diagram (Relation - Not a Function):
The relation is R = {(1, 4), (1, 5), (2, 5),(3,5)}
Element '1' is mapped to more than one element in B. This violates the rule that each input must have only one output.
Mathematically: $\exists a \in A, b, c \in B$ with $b \neq c$ such that $(a, b), (a, c) \in R$

Thus, only the first relation satisfies the definition of a function, while the second and third do not.

---

6. Key Differences Between Relation and Function

Aspect	Relation	Function
Definition	A subset of A × B	A relation with each input having one output
Input Mapping	One input can map to multiple outputs	One input maps to exactly one output
Example	{(1, 4), (1, 5),(2,5)} All three diagrams are relations	{(1, 4), (2, 4),(3,5)} Only First diagram fulfill condition of function.
Condition	No specific condition	Must pass the vertical line test

---

7. Examples of Relations and Functions

Relation but Not Function:
R = {(1, 4), (1, 5), (2, 5)}
This is not a function because the input '1' maps to two different outputs.

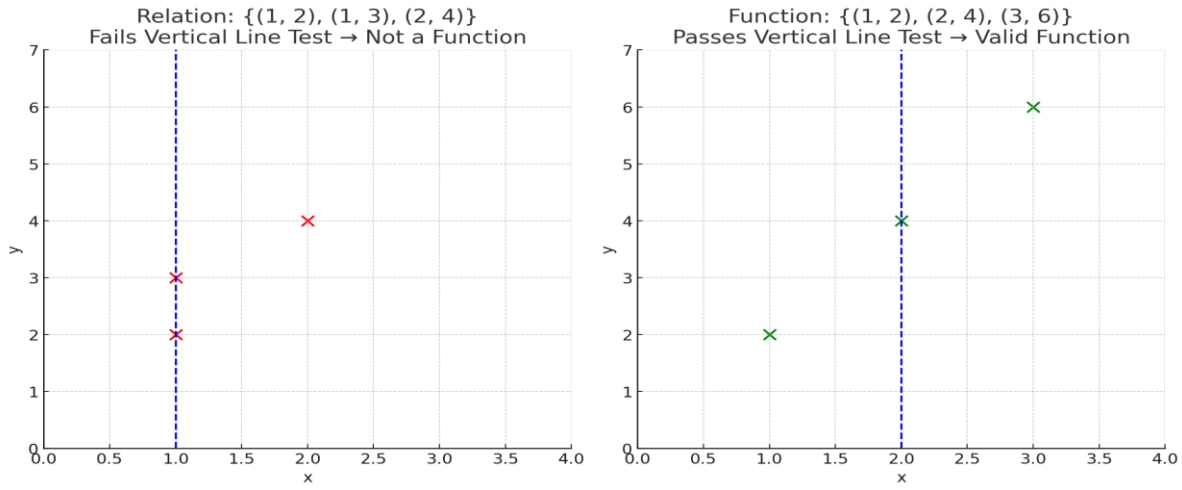
Relation is a Function:
f = {(1, 4), (2, 4), (3, 5)}
Each input has exactly one output → Valid function.

8. Graphical Illustrations (Use of Vertical Line Test in this Project)

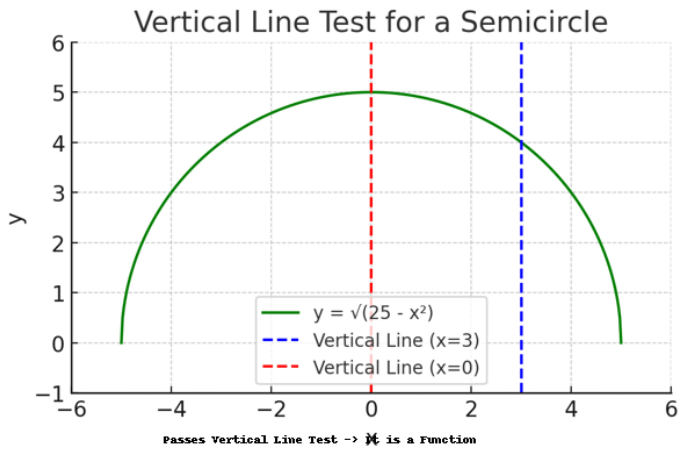
The Vertical Line Test (VLT) has been extensively used in this project to clearly distinguish between relations and functions. It provides a visual and mathematical method to determine whether a graph represents a function.

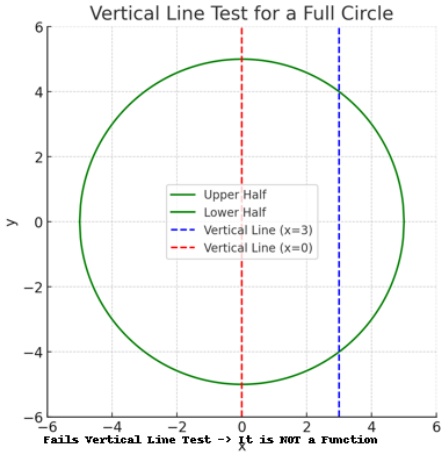
A. Graph of a Relation (Not a Function):
Consider the relation R = {(1, 2), (1, 3), (2, 4)}.
When plotted on a graph, two points exist vertically above x = 1.
Hence, it fails the Vertical Line Test → Not a function.

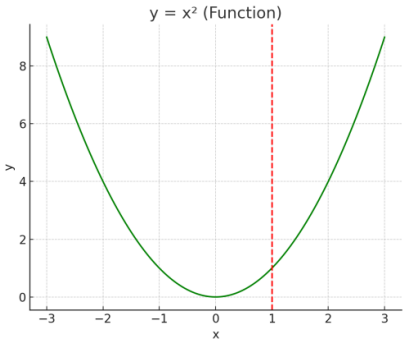
B. Graph of a Function:
Consider f = {(1, 2), (2, 4), (3, 6)}.
Each x-value has only one y-value.
Passes the Vertical Line Test → Valid function.

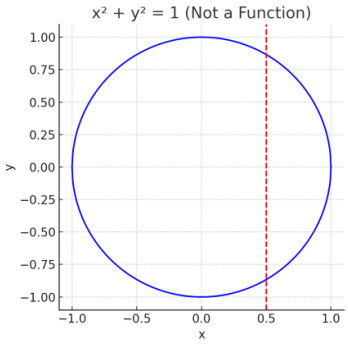


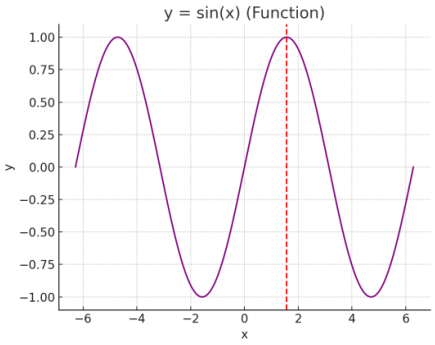
the concept becomes intuitive and easy to understand. VLT helps identify when an input value is associated with multiple outputs, which is not allowed in functions. Therefore, the use of the Vertical Line Test in this project is highly appropriate, as it supports the core objective — distinguishing relations from functions.











Additional Graphical Observations Using VLT

- Parabola (e.g., $y = x^2$) – Passes Vertical Line Test → It is a Function
- Semicircle (e.g., $y = \sqrt{(r^2 - x^2)}$) – Passes Vertical Line Test → It is a Function
- Full Circle (e.g., $x^2 + y^2 = r^2$) – Fails Vertical Line Test → Not a Function

9. Real-Life Applications

• Relation: A student can be enrolled in multiple courses – this represents a relation.
• Function: Assigning a unique roll number to each student – this represents a function.

10. Conclusion

To conclude, while both relations and functions deal with the association between two sets, functions are more structured and follow a specific rule. Graphical representation and the use of the Vertical Line Test further help to identify and understand the distinction between them in a clear and visual way.

11. References

1. ISC Class 11 Mathematics Textbook by ML Aggarwal
2. Concise Mathematics by R. D. Sharma
3. www.ncert.nic.in

Teacher's Remark:
(Signature)
(Name of Teacher)
Date: _____